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Histogram Equalization

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Outline

- Image Histogram
- Equalization Algorithm
- Theorem and Proof

Image Histogram

- Definition
  - The histogram of a digital image is a discrete function $p(r_k)$ with gray values $r_0, r_1, \dots r_{L-1}$.

$$p(r_k) = \frac{n_k}{n}$$

    - n_k: number of pixels with gray value r_k:
    - n: total number of pixels in the image

Image Histogram

- Definition
  - **Example:** A 10x10 image I_{in} with gray values with gray values $r_0 = 0, r_1 = 1, \ldots r_7 = 7$

0	6	5	3	4	1	1	3	4	1
3	2	3	4	3	2	3	4	3	2
2	1	0	3	4	2	2	0	4	2
1	0	5	6	1	2	5	6	7	2
6	3	4	6	1	1	4	6	7	1
3	1	5	2	1	3	0	2	1	3
2	1	2	0	4	2	2	3	4	2
1	0	5	6	7	2	5	6	7	2
1	3	4	1	7	0	4	0	7	1
3	1	5	2	1	3	5	2	1	0

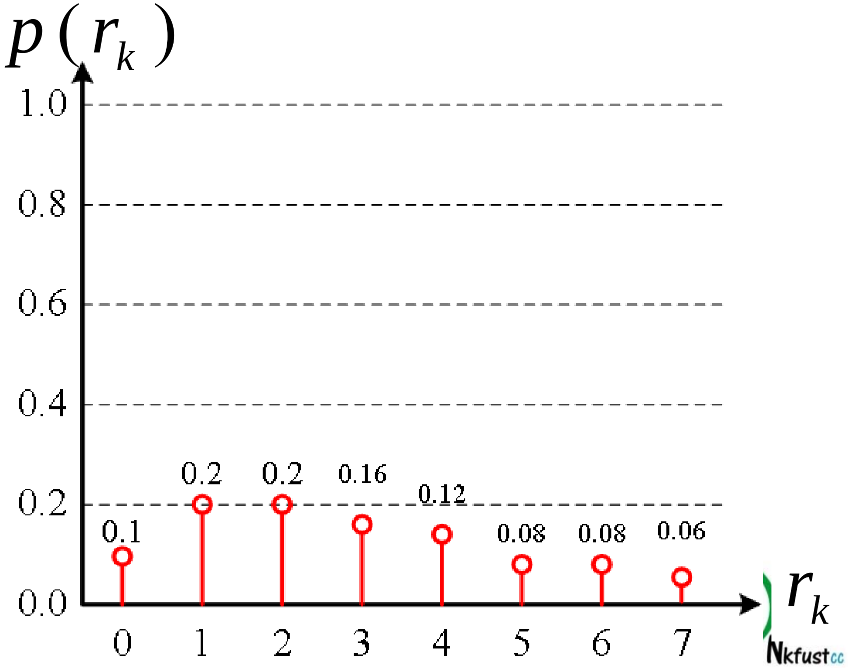


Image Histogram

- Typical Histograms
  - Histogram provides a global description of the image appearance

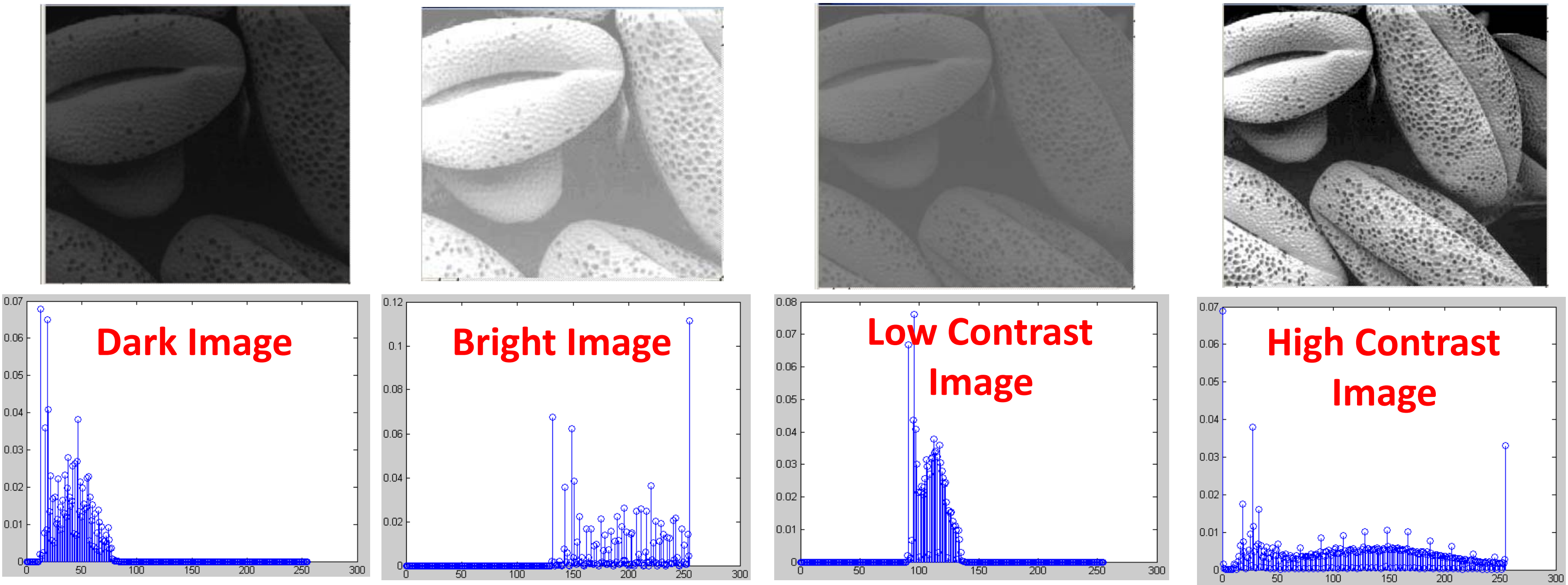
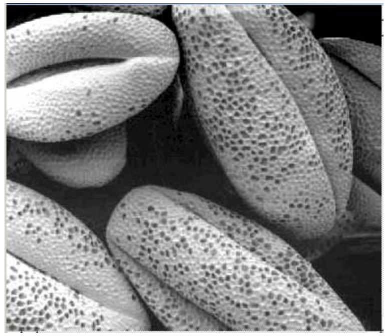
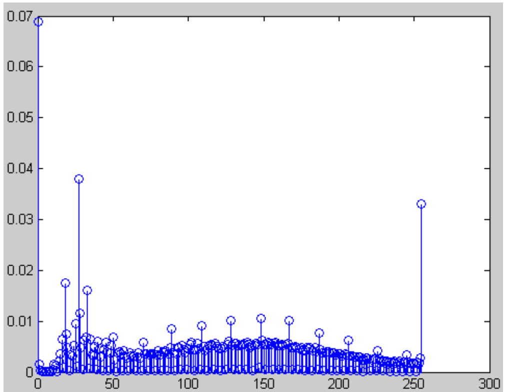


Image Histogram

- Typical Histograms

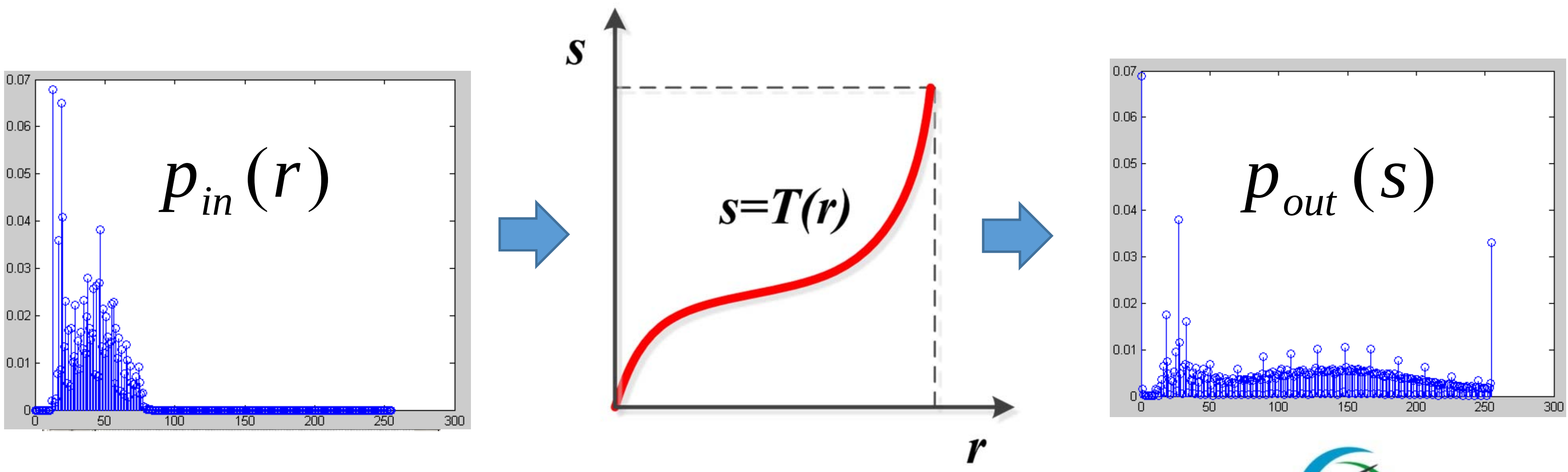




  - The histogram of the high contrast image is more uniformly distributed in all gray values.
  - Histogram equalization is an algorithm to transfer the original histogram to a more flat one.

Equalization Algorithm

- Formal Description
  - Design a transformation $s = T(r)$ to equalize the distribution of the original image I_{in}.



Equalization Algorithm

- Design of $s = T(r)$

Maximal Intensity

Cumulative Distribution Function

$$s_k = T(r_k) = (L-1) \times \sum_{j=0}^{k} p_{in}(r_j)$$

$$\boldsymbol{s_0 = T(r_0 = 0) = (L-1) \times} \sum_{j=0}^{k=0} p_{in}(r_j) = p_{in}(r_0)$$

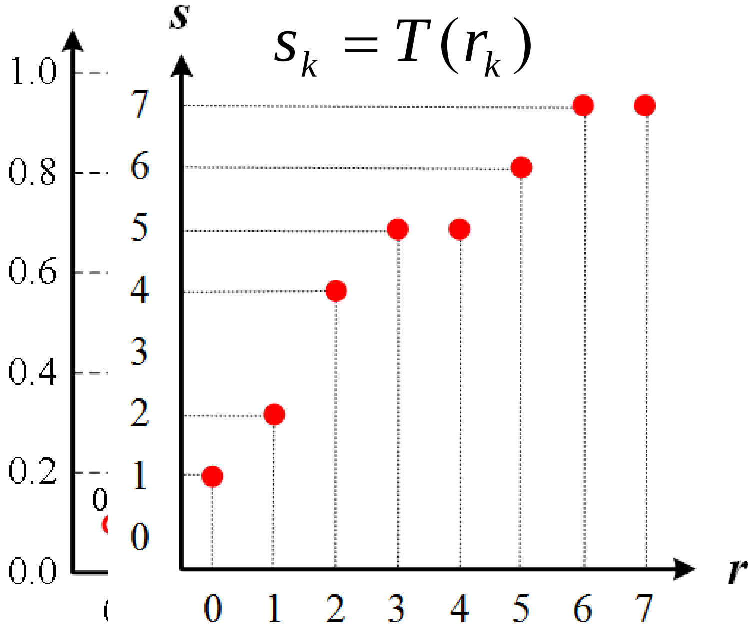
$$s_1 = T(r_1 = 1) = (L-1) \times \sum_{j=0}^{k=1} p_{in}(r_j) = (p_{in}(r_0) + p_{in}(r_1))$$

⋮ ⋮

Equalization Algorithm

- Design of $s = T(r)$

$$s_k = T(r_k) = (L-1) \times \sum_{j=0}^{k} p_{in}(r_j)$$



$$s_0 = T(r_0 = 0) = 7 \times (0.1) = 0.7 \rightarrow 1$$

$$s_1 = T(r_1 = 1) = 7 \times (0.1 + 0.2) = 2.1 \rightarrow 2$$

⋮

$$s_7 = T(r_7 = 7) = 7 \times (0.1 + \ldots + 0.06) = 7$$

$=1$

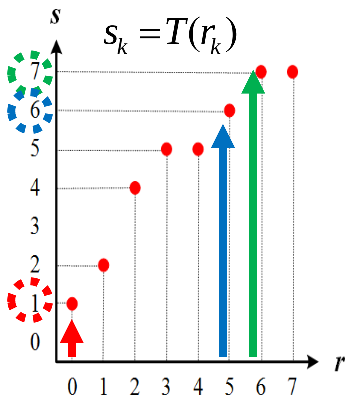
Equalization Algorithm

- Image Transformation

$$I_{out}(x,y) = T(I_{in}(x,y))$$

0	6	5	3	4	1	1	3	4	1
3	2	3	4	3	2	3	4	3	2
2	1	0	3	4	2	2	0	4	2
1	0	5	6	1	2	5	6	7	2
6	3	4	6	1	1	4	6	7	1
3	1	5	2	1	3	0	2	1	3
2	1	2	0	4	2	2	3	4	2
1	0	5	6	7	2	5	6	7	2
1	3	4	1	7	0	4	0	7	1
3	1	5	2	1	3	5	2	1	0

I_{in}

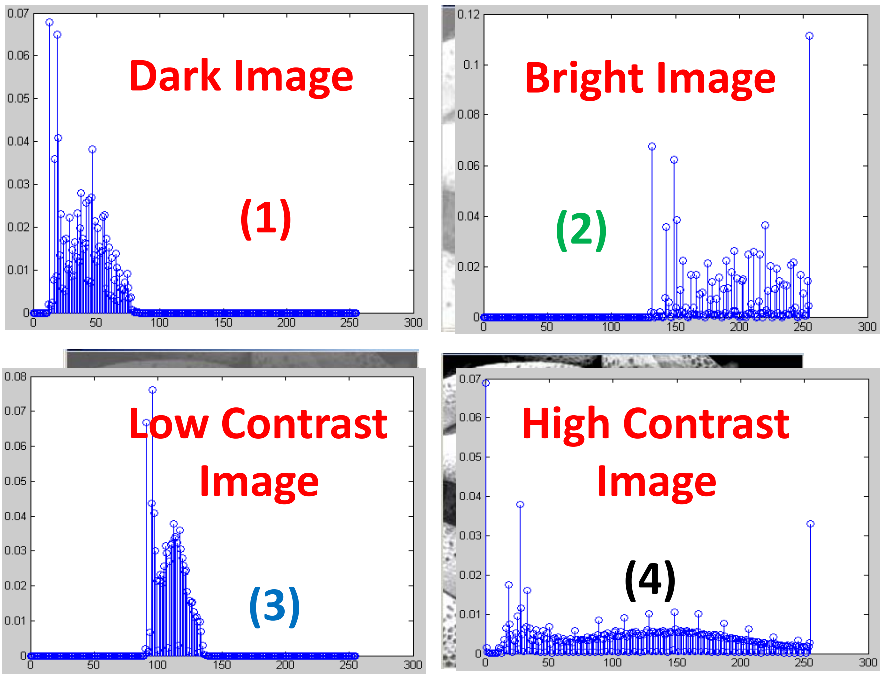


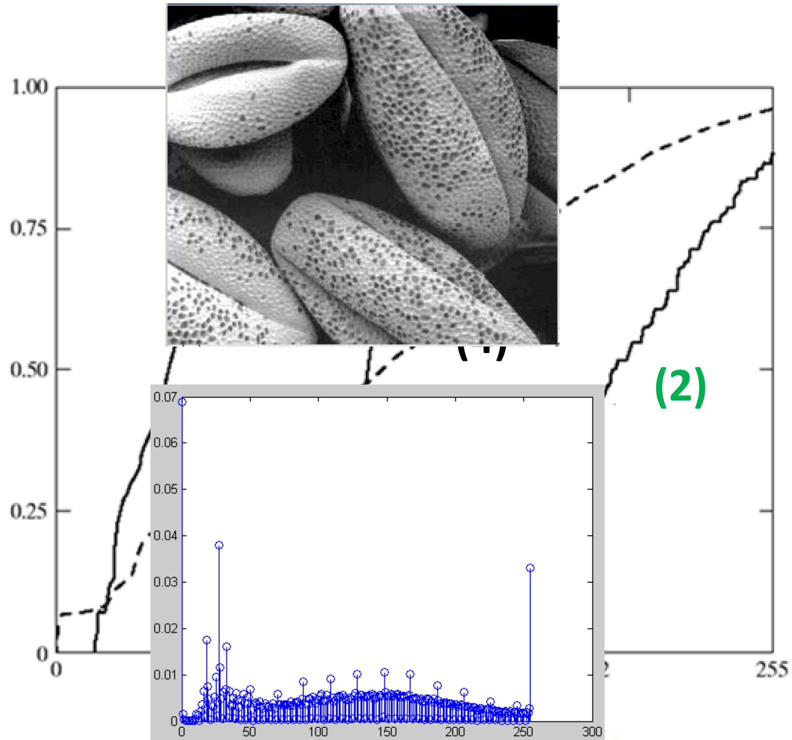
1	7	6	5	5	2	2	5	5	2
5	4	5	5	5	4	5	5	5	4
4	2	1	5	5	4	4	1	5	4
2	1	6	7	2	4	6	7	7	4
7	5	5	7	2	2	5	7	7	2
5	2	6	4	2	5	1	4	2	5
4	2	4	1	5	4	4	5	5	4
2	1	6	7	7	4	6	7	7	4
2	5	5	2	7	1	5	1	7	2
5	2	6	4	2	5	6	4	2	1

I_{out}

Equalization Algorithm

- Examples





Equalization Algorithm

- Pseudo Code
  - **Step 1:** Compute the histogram of input image

$$p(r_k) = \frac{n_k}{n}$$

  - **Step 2:** Compute the transformation function

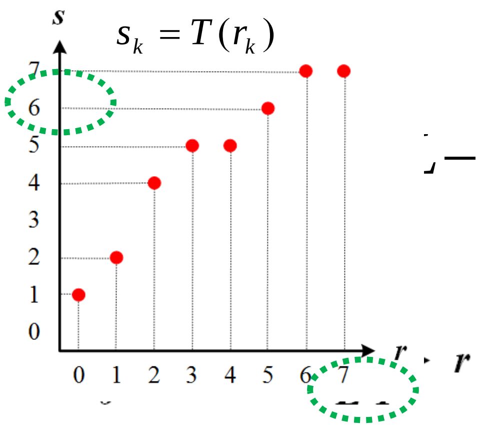
$$s_k = T(r_k) = (L-1) \times \sum_{j=0}^{k} p_{in}(r_j)$$

  - **Step 3:** Transform the value of each pixel by

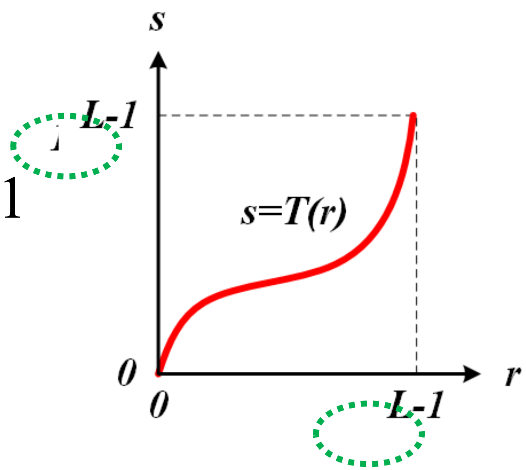
$$I_{out}(x,y) = T(I_{in}(x,y))$$

Theorem and Proof

- Assumptions
  - *T(r)* is a **monotonically increasing continuous** function without loss of generality
  - *T(r)* is a function mapping from [0,1] to [0,1]



$L-1 \Rightarrow 1$



Theorem and Proof

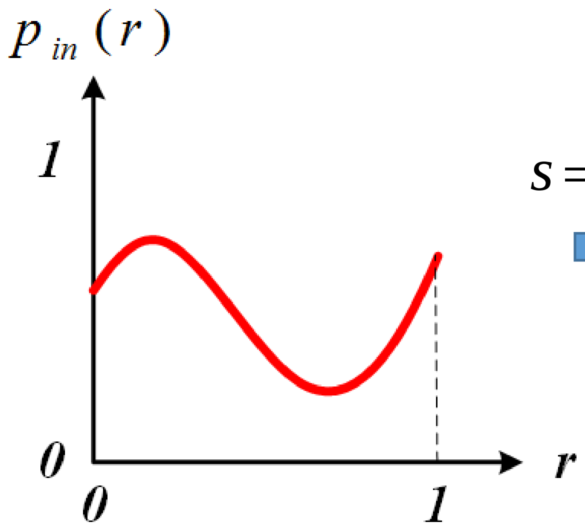
- Proposition
  - Let $p_{in}(r)$ denote the probability density of the Gray values in the input image.
  - Let $p_{out}(s)$ be the probability density of $p_{in}(r)$ after transformation $s = T(r)$

Discrete: $s_k = T(r_k) = \sum_{j=0}^{k} p_{in}(r_j)$

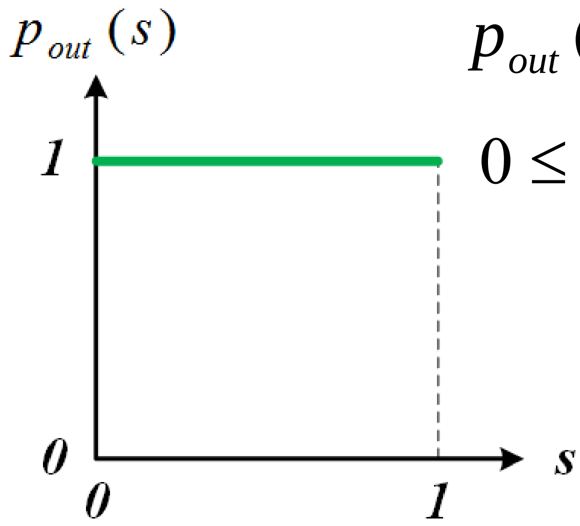
Continuous: $s = T(r) = \int_0^r p_{in}(w)dw \quad 0 \le r \le 1$

Theorem and Proof

- Proposition $s = T(r) = \int_0^r p_{in}(w)dw$
  - **Prove:** $p_{out}(s)$ is uniformly distributed in$[0,1]$



$s = T(r)$

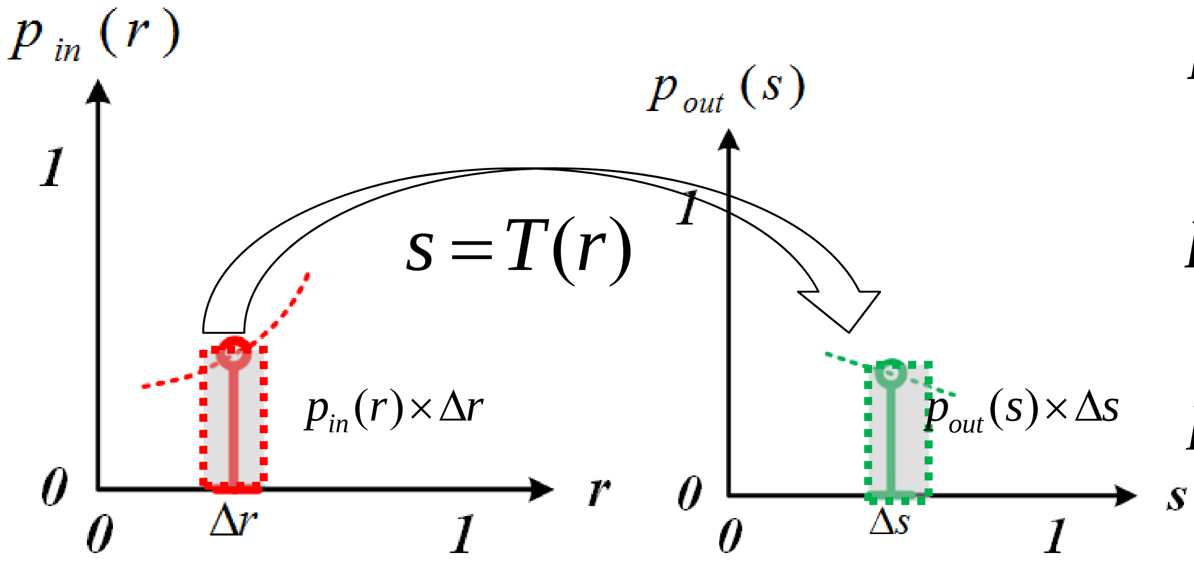


$p_{out}(s) = 1$

$0 \le s \le 1$

Theorem and Proof

- Proof
  - Use the fundamental theorem of calculus



$$p_{out}(s)\times\Delta s = p_{in}(r)\times\Delta r$$

$$\Downarrow$$

$$p_{out}(s)\times ds = p_{in}(r)\times dr$$

$$\Downarrow$$

$$p_{out}(s) = p_{in}(r)\frac{dr}{ds}$$

Theorem and Proof

- Proof
  - Derive $p_{out}(s)$ using $s = T(r) = \int_0^r p_{in}(w)dw$

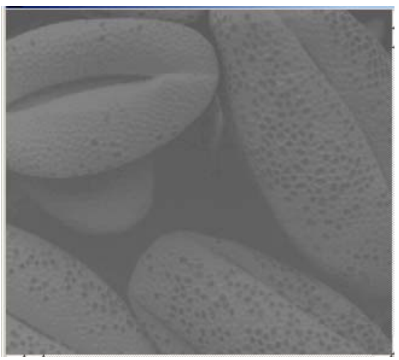
$$p_{out}(s) = p_{in}(r)\frac{dr}{ds} = p_{in}(r)\left(\frac{ds}{dr}\right)^{-1}$$

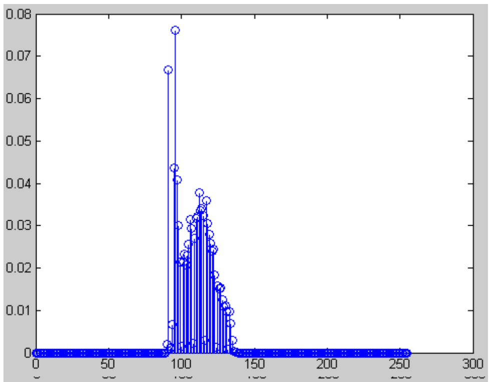
$$= p_{in}(r)\left(\frac{d\int_0^r p_{in}(w)dw}{dr}\right)^{-1} = p_{in}(r)\left(p_{in}(r)\right)^{-1} = 1$$

⇒ $p_{out}(s) = 1$ **Uniformly Distributed**

Theorem and Proof

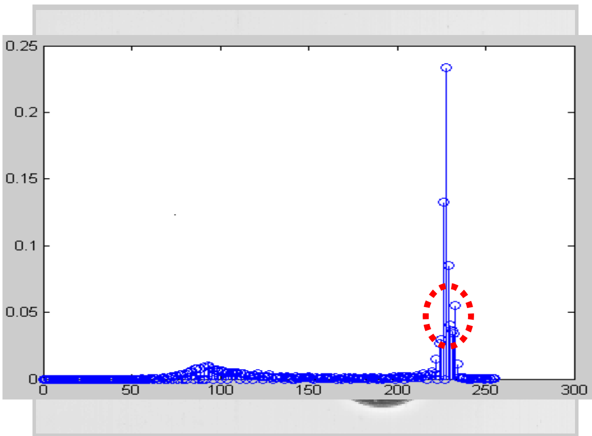
- Discussion
  - The output probability density $p_{out}(s)=1$ is uniform regardless of the input
  - The histogram of output image is only approximately uniform since it is in the **discrete** case.





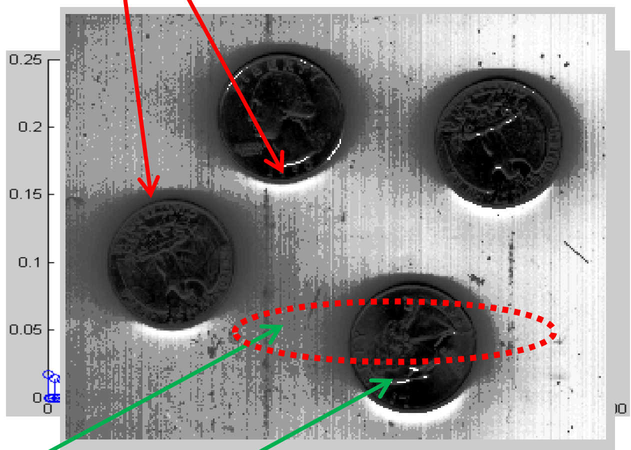
Theorem and Proof

- Discussion
  - This algorithm usually **but not always** results in an enhanced image.



Histogram Equalization

false edges



“graininess” and “patchiness”


thank you
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